Written exam in the study of Economics

LINEAR MODELS

Monday, June 27, 2011.

(3 hours written exam. All ususal aids allowed (i.e. books, notes etc.), but it is neither allowed bringing any electronic calculator nor using any other electronic equipment. Open book exam.)

The language of the exam is English.

KØBENHAVNS UNIVERSITETS ØKONOMISKE INSTITUT

2011Su-2LM ex

Examination in Linear Models

Monday, June 27, 2011.

This is a 3 hour examination (2 pages with a total of 4 exercises).

It is allowed using textbooks, notes and personal notes. It is strictly prohibited using calculators or cas tools.

Problem 1. In $\mathbf{R}^4$ are given the vectors $u_1 = (1, 0, -1, 0), u_2 = (0, 2, 1, 1)$ and $u_3 = (1, -4, -3, -2)$. We denote span$\{u_1, u_2, u_3\} = U$.

(1) Show that span$\{u_1, u_2, u_3\}$ = span$\{u_1, u_2\}$.

(2) Determine the coordinates of u_3 with respect to the basis u_1, u_2 in U.

(3) Let the linear mapping $T : U \to U$ be given by $Tu_1 = u_1 + u_2$, $Tu_3 = u_1 - u_2$. Determine the matrix corresponding to T with respect to the basis u_1, u_2 in U.

(4) Show that T is invertible, and find the matrix for the inverse, T^{-1}.

(5) Show that u_2 is an eigenvector for T, and find the corresponding eigenvalue.

Problem 2.

The $3 \times 3-$matrix A has the eigenvalues $1, 2$ and 3, with corresponding eigenvectors $(1, 0, 1), (0, 1, 0)$ and $(-1, 0, 1)$.

(1) Determine one of the possible versions of the matrix A.

(2) Find the matrix $\ln(A)$ (where ln denotes the natural logarithm).

(3) Show that A is invertible and that $\ln(A)$ is not invertible.

Problem 3.

(1) Calculate the integral $\int \cos^2(2x) \sin(3x)dx$.

(2) Solve the equation $2z^2 - 4z + 4 = 0$.

Problem 4.
 We consider the series

$$\sum_{n=0}^{\infty} \left(\frac{2}{3 - \cos(x)}\right)^n.$$

(1) Show that the series is convergent for $x \neq p2\pi$, $p \in \mathbf{Z}$.

(2) Find an expression for the function $f(x) = \sum_{n=0}^{\infty} \left(\frac{2}{3-\cos(x)}\right)^n$.

(3) Determine the range, $R(f)$, for the function f.